

# Characteristics of tides in open oceans, modelling of tides (global and regional)

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# Tidal propagation

- Progressive waves
- Standing waves
- Rotational effects of the earth

# Co-tidal lines and co-range lines

- Lines of equal tidal range are called co-tidal lines and lines of equal phase are called co-range lines

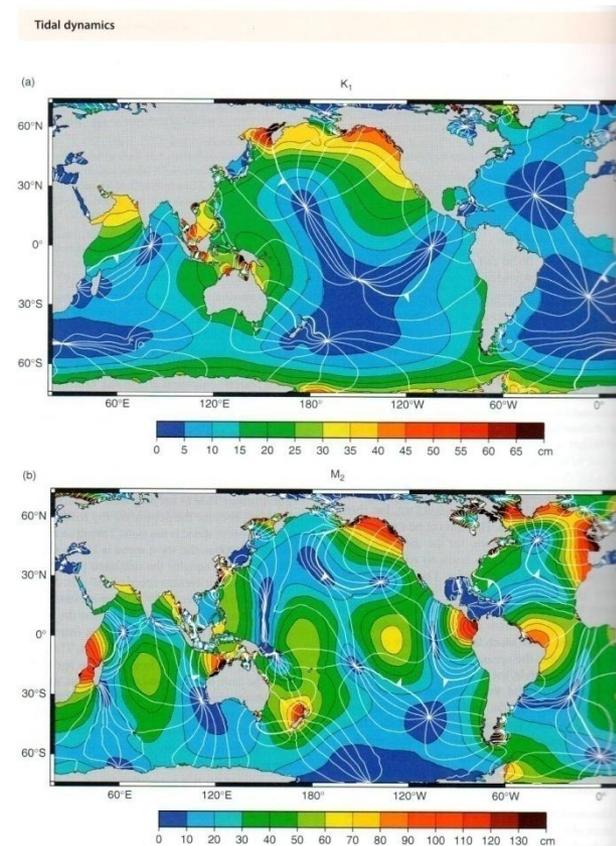
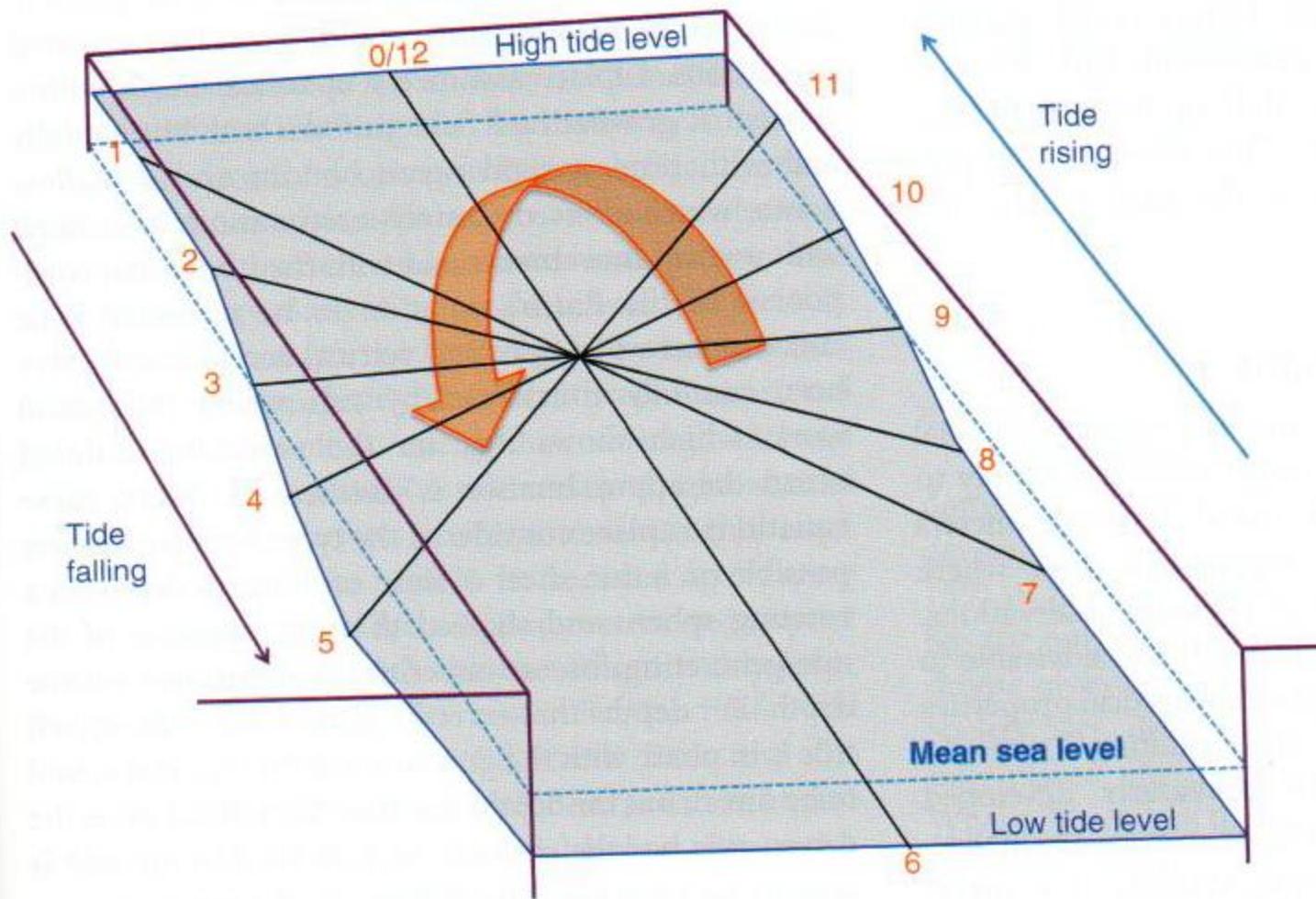


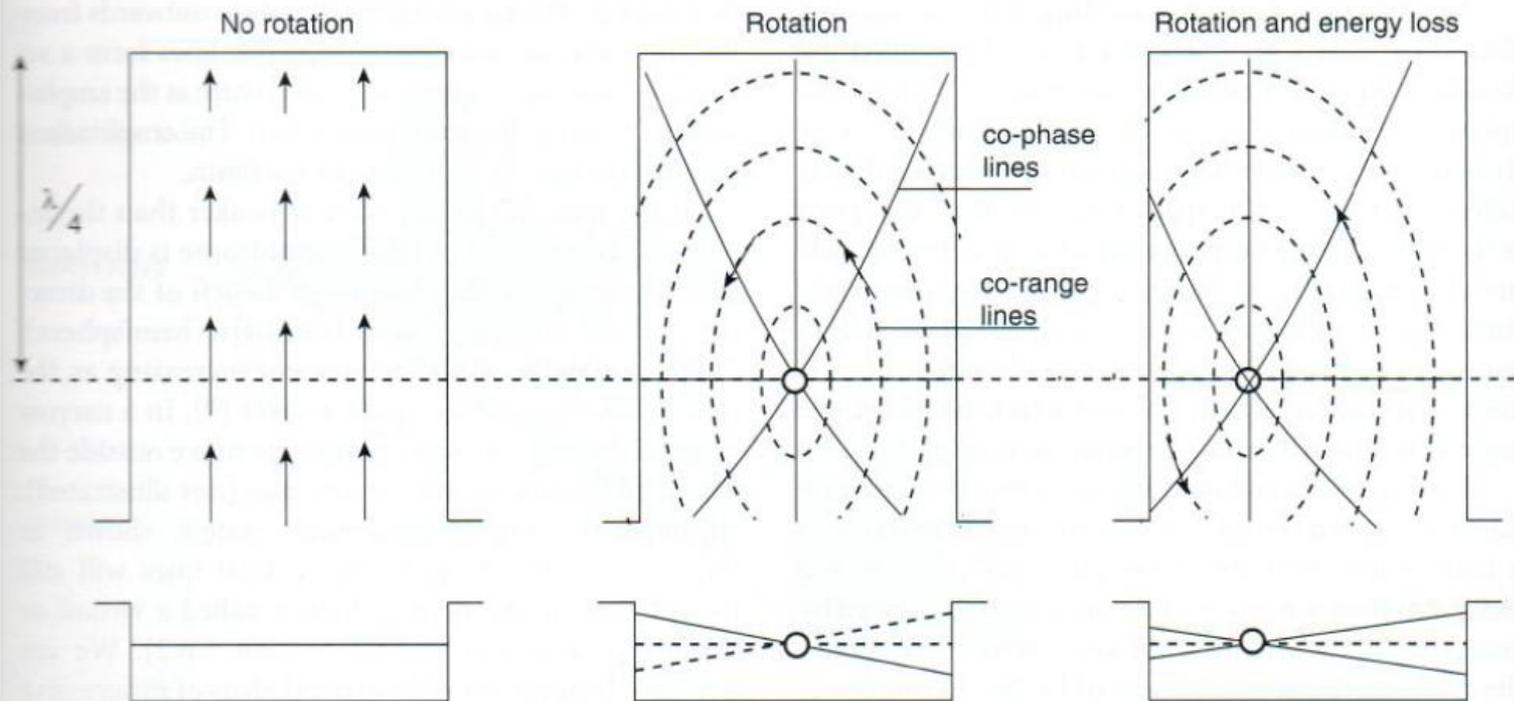
Figure 5.1 Co-tidal charts of the ocean tide: global maps of lines joining places where high tides for (a)  $K_1$  and (b)  $M_2$  occur simultaneously.

# Amphidromic points

- Amphidromic points are the characteristic feature of tidal propagation. Regions where tidal ranges are nearly zero, but tidal currents can be present.
- Points where the tidal waves rotate due to the Coriolis force and basin effects.
- Example: M2 amphidromic points are found off Sri Lanka, southern part of the Arabian Sea

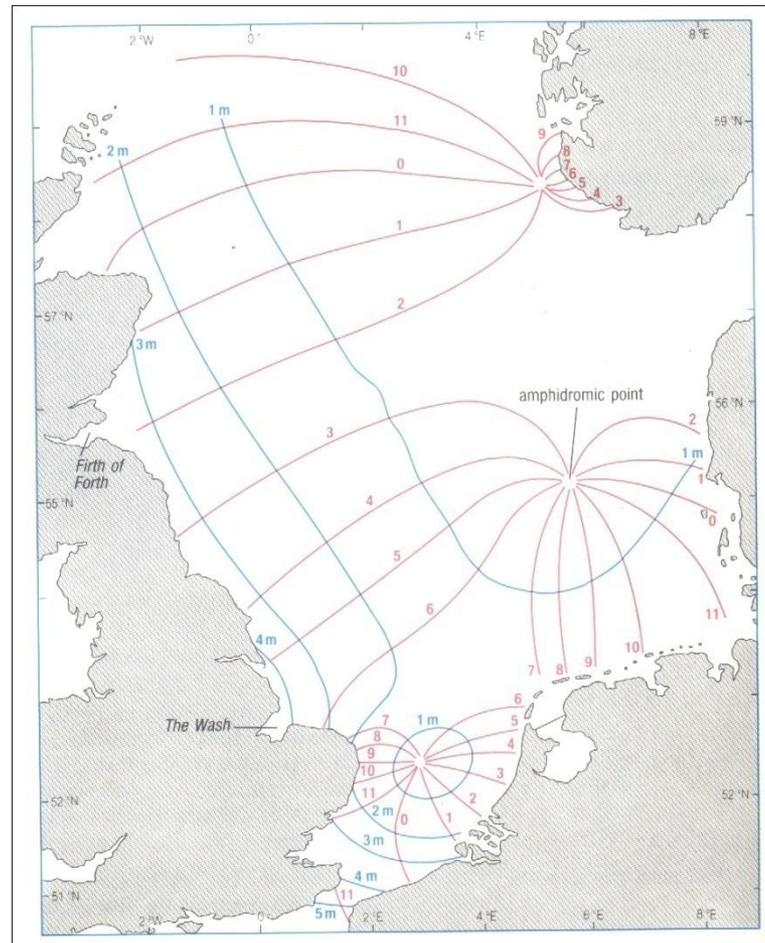


**Figure 5.7** Amphidrome dynamics: a three-dimensional drawing exaggerated to illustrate how a tidal wave progresses around an amphidrome in a basin in the northern hemisphere. The red numbers are hours in the semidiurnal cycle.



**Figure 5.8** Amphidrome dynamics: the effects of the Earth's rotation on a standing wave in a basin that is slightly longer than a quarter wavelength. With no rotation there is a line of zero tidal amplitude. Because of the Earth's rotation, the tidal wave rotates around a point of zero amplitude, called an amphidromic point. In the third case, because the reflected wave has lost energy through tidal friction, the amphidrome is displaced from the centre line.

# Amphidromic points in the North Sea



# Regions of extra-ordinary tidal ranges

- Bay of Fundy- Largest observed tides
- Gulf of St Malo - Largest observed M2 tides
- Gulf of Kachh and the Gulf of Khambhat – Largest tides in the north Indian Ocean coasts.

Geometric amplification, quarter-wave length resonance etc. are responsible for high amplification

# Merian's formula

- Merian's formula for the fundamental period of oscillation of a bay is
- $T=2l/n\sqrt{gh}$
- For instance, a rough application of this formula gives, For a bay of length  $\sim 200$  km, for  $n=1$  (mode) and an average depth of 10 m, this comes to about  $\sim 11$  hours, close to the semi-diurnal period
- In the Gulf of Kaachh, the amplitudes of M2 increase nearly two-fold from the mouth (Okha) to the head (Navalkhi/Kandla), whereas the amplification of K1 or O1 is only marginal ( $\sim 1.2$  times)

Equations of motion can be written as

$$\frac{D\mathbf{u}}{Dt} + 2\boldsymbol{\Omega} \times \mathbf{u} = -\frac{1}{\rho} \nabla P - g - \nabla \phi_T$$

where  $\phi_T$  is the tidal potential

for tides, as wave lengths are large when compared to the ocean depths, shallow-water

approximations can be used.

the approximations can be used

- Shallow water equations can be used, as the wave lengths are large compared to ocean depth. Often, they are known as Laplace's tidal equations. They contain an additional term, to account for the tidal potential. These equations though developed originally for spherical co-ordinates, can be expressed in Cartesian ... The errors involved are small.

The equations of motion, when the tide-producing force is included, have an additional force per unit mass  $-\nabla\Phi_T$  on the right-hand side of the momentum equation (4.10.11), which becomes

$$D\mathbf{u}/Dt + 2\boldsymbol{\Omega} \times \mathbf{u} = -\rho^{-1} \nabla p - \mathbf{g} - \nabla\Phi_T. \quad (9.8.2)$$

However, since the horizontal scale of the forcing is very large compared with the depth, the shallow-water approximation can be used, as it was by Laplace. Now tides are a global phenomenon and of global scale, so it may seem inappropriate to discuss them at this stage when only the “ $f$ -plane approximation” (see Section 7.4) to the equations has been introduced, this approximation being appropriate to motions with scales that are small compared with the radius of the earth. However, the semi-

TABLE 9.1

Principal Constituents<sup>a</sup> of the Tides (Those with Amplification Factor  $> 0.1$ )<sup>b</sup>

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# Tidal equations

water equations Laplace's law

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - fv = -g \frac{\partial (\eta - \eta_e)}{\partial x}$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + fv = -g \frac{\partial (\eta - \eta_e)}{\partial y}$$

$$\eta_e = -\frac{\Delta T}{g}$$

$f$  Coriolis parameter  
 $g$  accel. gravity

$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x} [(H + \eta)u] + \frac{\partial}{\partial y} [(H + \eta)v] = 0$$

- Assumptions:
- Tides are barotropic
- However, internal tides can be generated through interaction of barotropic tides with topographic features (continental slope, ridge etc..). It is assumed that baroclinic tides do not get affected by interaction.
- Bottom friction can affect (reduce) barotropic currents in the bottom.

# Solid earth tides

They can be determined

- $H = -(1+k-h) \phi_T/g = -0.7 \phi_T/g$
- $k, h$  are Love numbers
- Effectively, solid earth tides can reduce ocean tides by 30 %. There are two processes. (i) Elastic response causes a bulge with an elevation of  $h \phi/g$ , where  $h$  is constant ( $\sim 0.6$ )  
Surface elevation in the ocean will be reduced by this much

(ii) Besides, the bulge on the solid earth will change gravitation. Additional gravitational potential due to the bulge is  $k \phi_T$  where  $k$  is about 0.3

Since  $\eta$  is measured relative to the solid earth, it is necessary to replace  $\eta$  in the continuity equation by  $\eta - \phi_T/g$

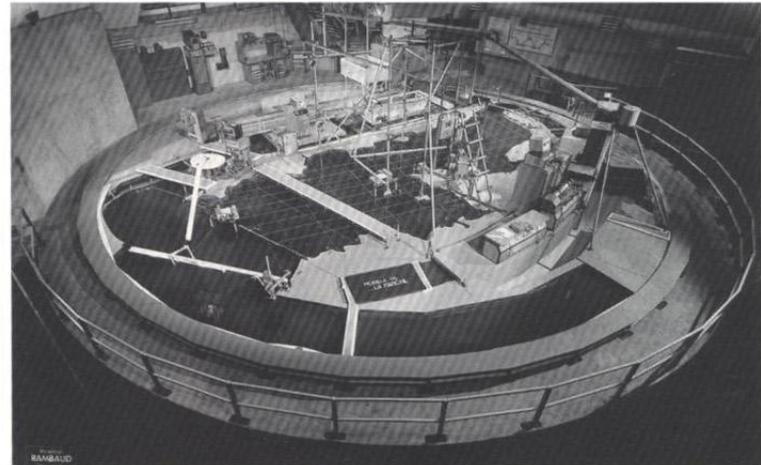
# Tidal modelling

- Chabert d'Hiers and Le Provost (1978) – Physical model for the English Channel
- Schwiderki's model
- Global tidal models since the altimeter era

# Physical Model

- Developed in Grenoble (Chabert d'Hiers and Le Provost, 1978) for simulating tides in English Channel
- (Fig. taken from Cartwright)

*Tide models for the world ocean*



**Figure 12.3.** The 'Coriolis' Rotating Platform of the University of Grenoble, as used with a hydraulic model of the tides in La Manche (English Channel). With a diameter of 14 meters, the model had a horizontal scale of 1/50,000 and a vertical scale of 1/500, corresponding to a time scale of 1/2,250. Rotation at a stable 50.4 seconds period, representing  $(2\pi/\text{Coriolis frequency})$ , simulated the Coriolis stress at the appropriate latitude. The scaled period of the  $M_2$  tide was 20 seconds. Since 1987, the platform has carried a cylindrical tank of 13 meters diameter, used to simulate internal waves with rotation. (Photograph courtesy of Gabriel Chabert-d'Hières.)

# Global tidal models

- Schwiderski's model – 1 deg resolution
- Used observed coastal and deep sea tide gauges
- Aimed at an accuracy of 10 cm in amplitude and 5 degree in phase for major tidal constituents

## Global tidal models- Assimilation of altimeter data

- Global tidal models were developed by mid-1990's, with the assimilation of satellite altimeter data
- While global tidal models provide information on tides accurately in deep-sea, near the coast ( $< 20$  km), the accuracy is not sufficient.
- In order to overcome this, there are two ways (i) assimilate tide gauge data (Matsumoto ,2000) or (ii) develop regional tidal models

# Regional tidal models

- Regional tidal models use information on tides from global tidal models to prescribe open boundary conditions.
- This helps in developing fine-resolution regional models covering smaller domains. Moreover, using a fine resolution near the coast helps to obtain more accurate results near the coast.

# Limitations of global tidal models in the north Indian Ocean

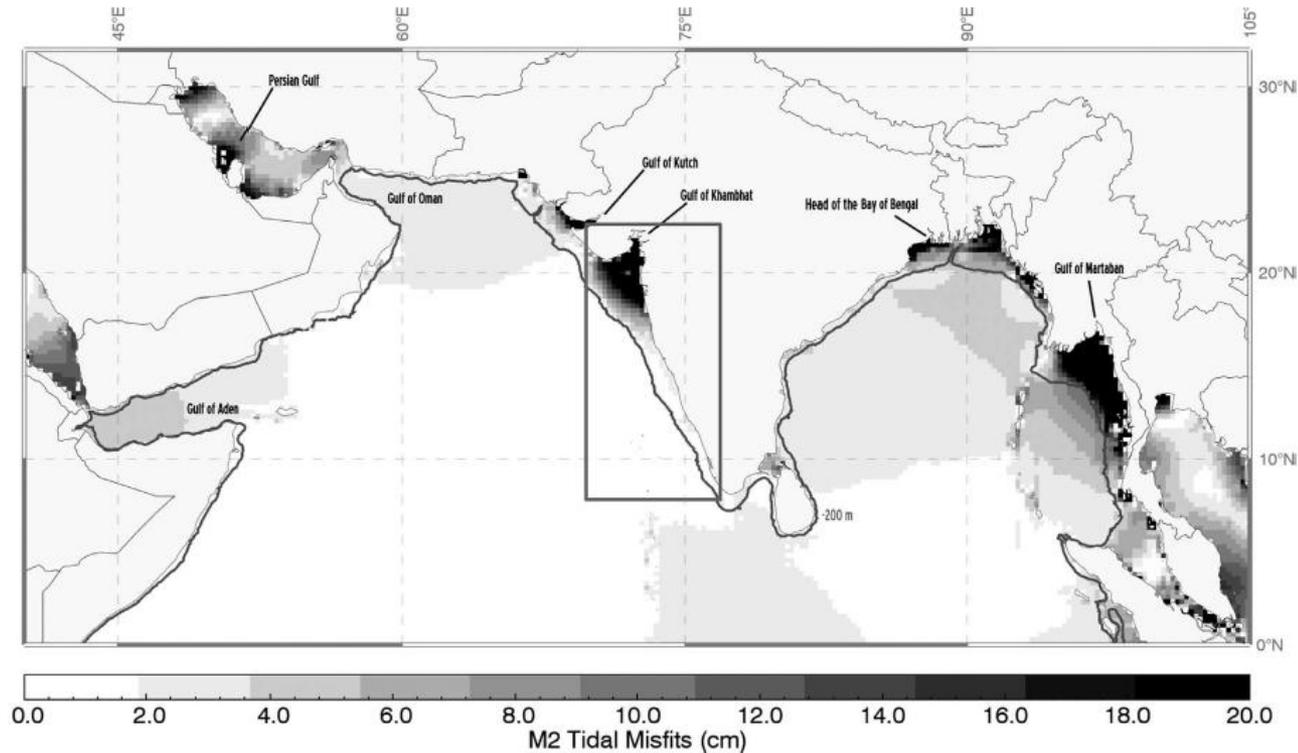


Figure 1. Map of the RMS misfits for M2 (cm), computed using Equation (2) between GOT4.7 and FES2012 hydrodynamic tidal solutions. The rectangle indicates the zone of interest of the present study, the West Indian coast.

– *In : Testut and Unnikrishnan 2016*

# Regional modelling of tides- Example off the west coast of India

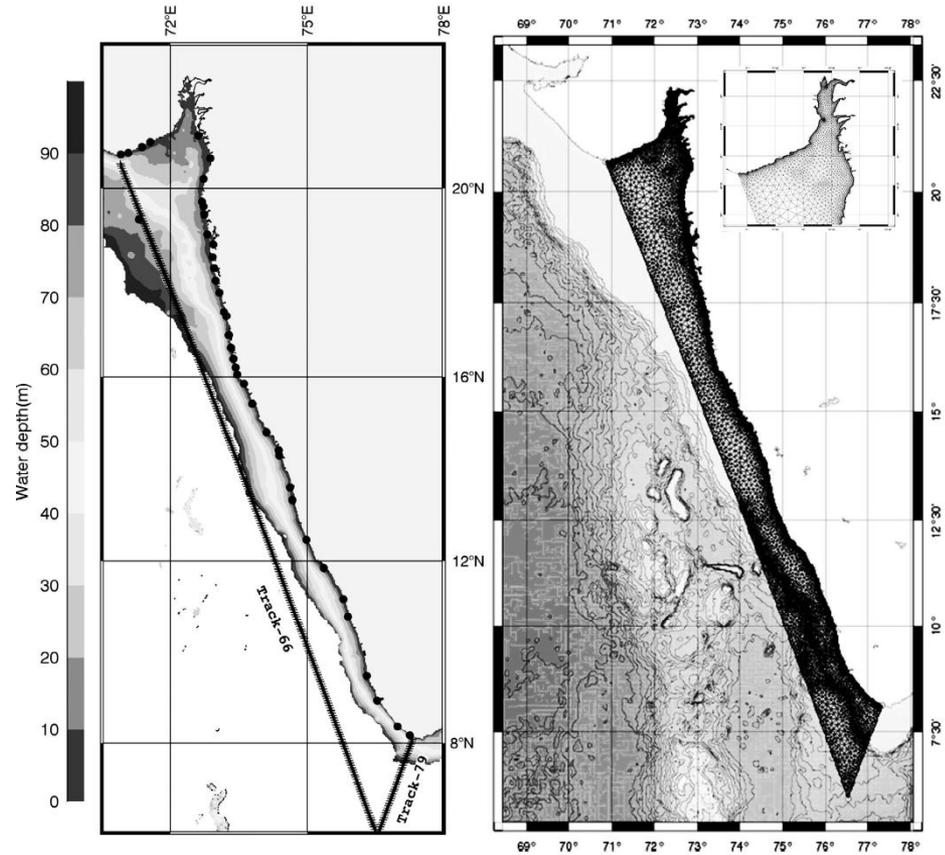


Figure 2. The left panel shows depth contour of the inner shelf bathymetry (0–100 m). The black dot indicates the position of tide gauges used for validation. The two tracks (66 and 79) used as boundary conditions are shown. Right panel shows the model mesh along the west coast of India. The embedded upper right panel is a zoom of the mesh inside the Gulf of Khambhat. The spatial resolution of the mesh ranges from less than 1 km at the coast to a maximum of 20 km in the open ocean. Bathymetric features outside the model domain are also shown.

# Tidal Simulations off the west coast of India

- M2 co-tidal and co-range maps

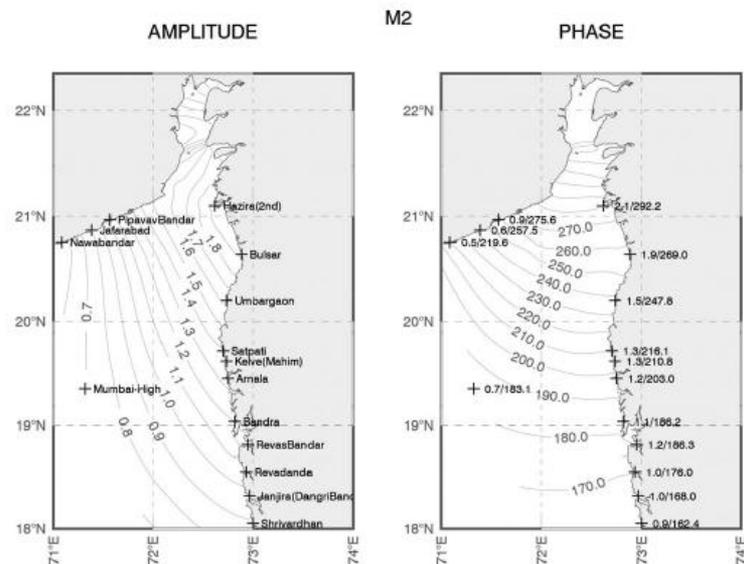


Figure 7. Map of amplitude  $A$  (left panel) and phase  $G$  (right panel) of the M2 constituent of the model in the region of the Gulf of Khambhat. The contours indicate amplitude in meters on the left panel and the phase in degrees on the right panel. Location and name of the used tide gauge are shown in the left panel, and their observed amplitude/phase in the right panel (in the same units as the contour lines).

# Tidal Simulations off the west coast of India

- K1 co-tidal and co-range maps

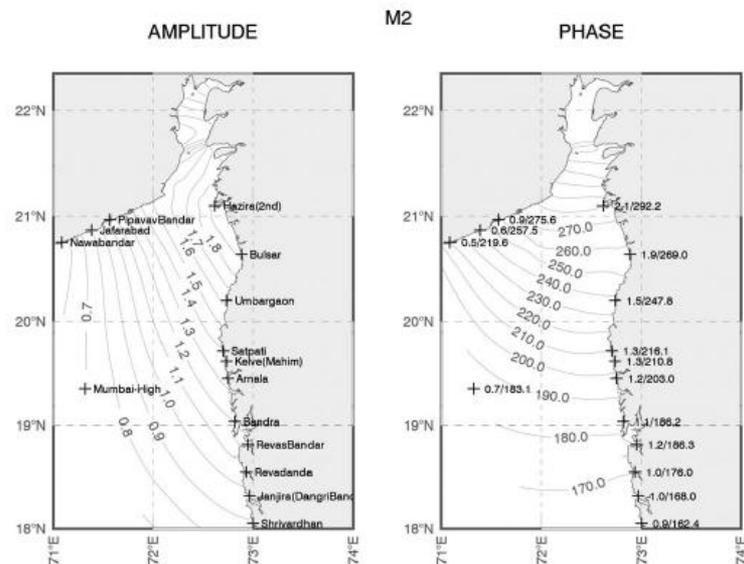


Figure 7. Map of amplitude  $A$  (left panel) and phase  $G$  (right panel) of the M2 constituent of the model in the region of the Gulf of Khambhat. The contours indicate amplitude in meters on the left panel and the phase in degrees on the right panel. Location and name of the used tide gauge are shown in the left panel, and their observed amplitude/phase in the right panel (in the same units as the contour lines).

# Characteristics of tides in the Bay of Bengal

M2 co-tidal and co-range lines  
Amphidrome south of SriLanka,  
degenerate amphidrome in the northern  
Bay  
amplification of M2, S2 in the head bay  
and both M2 and K1 amplification in the  
Malacca strait

*In : Sindhu and Unnikrishnan, 2013*

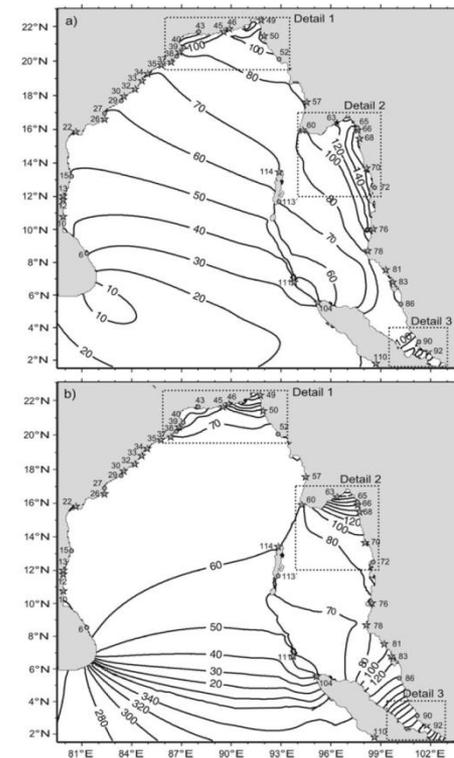


Figure 9. Distribution of amplitudes and phases of  $M_2$  tide in the Bay of Bengal simulated using the tidal model showing (a) co-range lines and (b) co-tidal lines. Amplitudes are given in centimetres and phase 'G' is given in degrees. The areas marked as Detail 1, Detail 2, and Detail 3 are shown in Figures 15, 16, and 17, respectively.

# Conclusions

- Regional tidal models provide accurate simulation of tides, if good bathymetry is used and proper boundary conditions are applied.